General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Monday 20 June 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables;
- an insert for use in Questions 3 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Answer **all** questions.

1 The scores of the five members of a sports quiz team on practice questions were as shown in the table.

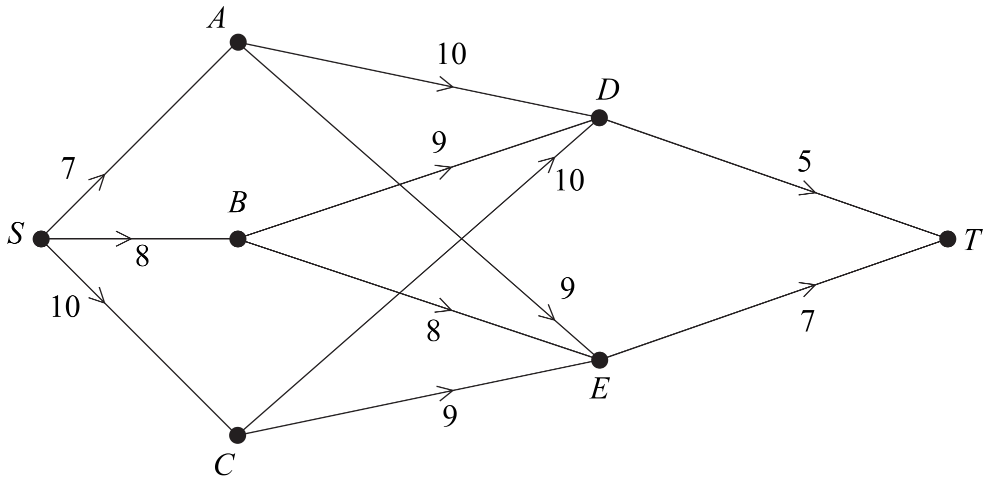
	Athletics	**Tennis**	**Football**	**Swimming**	**Golf**
Les	17	19	18	15	16
Mel	20	18	15	19	17
Nick	13	17	17	16	14
Ollie	12	16	18	15	14
Pete	14	16	15	16	15

A different person is to be chosen to answer questions on each one of the five sports so that the highest total score of the five team members is achieved.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $20 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table from 20. Use the Hungarian algorithm, reducing **rows first** and then columns, to allocate the sports to the team members in order to maximise the team's expected score on the basis of success at the practice questions. *(9 marks)*

2 A three-day journey is to be made from S to T, with overnight stops at the end of the first day at one of the locations A, B and C, and at the end of the second day at either D or E. The network shows the journey time, in hours, for each day of the journey.



The optimal route required is that in which the longest day's journey is as small as possible.

(a) Explain why the three-day route $SAET$ is better than $SADT$. *(2 marks)*

(b) Use dynamic programming to find the optimal three-day route from S to T. *(8 marks)*

TURN OVER FOR THE NEXT QUESTION

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

A cleaning project is to be undertaken. The table shows the activities involved.

Activity	**Immediate Predecessors**	**Duration (hours)**
A	—	4
B	—	6
C	*A*, *B*	7
D	*C*	9
E	*C*	10
F	*B*	3
G	*D*, *E*	6
H	*F*, *G*	5
I	*G*	3
J	*H*, *I*	2

(a) Complete an activity network for the project on **Figure 1**. *(4 marks)*

(b) Find the earliest start time for each activity. *(2 marks)*

(c) Find the latest finish time for each activity. *(2 marks)*

(d) (i) Write down the shortest time in which the whole cleaning project can be completed. *(1 mark)*

(ii) State which activities are the critical ones and explain how you can recognise them. *(3 marks)*

(e) A problem occurs which means that activity *I* cannot be started until 3 hours after activity *H* has started.

(i) Find the new earliest start time for activity *I*, giving a reason for your answer. *(2 marks)*

(ii) Find the revised shortest completion time for the project. *(2 marks)*

4 A linear programming problem involving variables x and y is to be solved. The objective function to be maximised is $P = 3x + 2y$. The initial Simplex tableau is given.

P	x	y	r	s	t	*value*
1	-3	-2	0	0	0	0
0	4	5	1	0	0	36
0	2	1	0	1	0	12
0	5	2	0	0	1	35

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, write down **three** inequalities in this problem. *(2 marks)*

(b) (i) Perform two iterations using the Simplex method, by initially choosing a value in the x-column as a pivot, to solve this linear programming problem. Comment on how you know that the optimum solution has been achieved. *(7 marks)*

(ii) State your final values of P, x and y. *(2 marks)*

(iii) State the values of the slack variables r, s and t at the optimum solution. *(1 mark)*

5 Two people, Pat and Quigley, play a zero sum game. The game is represented by the following pay-off matrix for Pat.

		Quigley		
	Strategy	**Q_1**	**Q_2**	**Q_3**
Pat	**P_1**	4	3	5
	P_2	-1	5	-2
	P_3	1	2	3

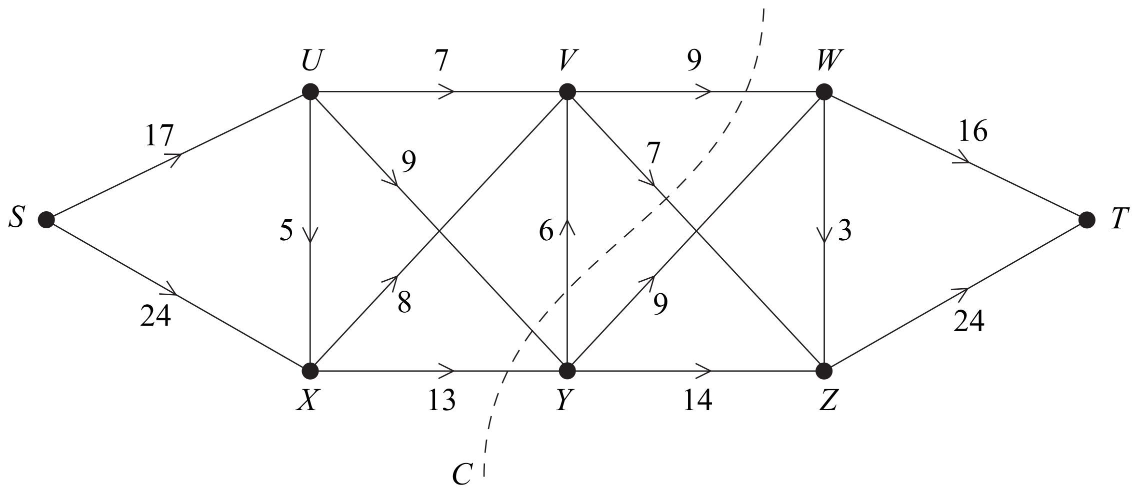
(a) Show that there is no stable solution. *(3 marks)*

(b) Explain why Pat should never play strategy P_3. *(1 mark)*

(c) Find the optimal mixed strategy for Pat. *(7 marks)*

6 [**Figures 2** and **3**, printed on the insert, are provided for use in this question.]

The diagram shows a network of corridors in a building. People move steadily along the corridors from the entrance S towards the exit T. The weights on the edges show the maximum number of people per minute that can move along each corridor.



(a) (i) Find the value of the cut marked C on the diagram. *(1 mark)*

(ii) Use your answer to part (a)(i) to make a deduction about the maximum flow of people through the building. *(2 marks)*

(b) State the maximum possible flows along the routes $SUYWT$ and $SXYZT$. *(2 marks)*

(c) (i) Taking your answers to part (b) as the initial flow, use a labelling procedure on **Figure 2** to find the maximum flow from S to T. *(6 marks)*

(ii) State the value of the maximum flow and, on **Figure 3**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(iii) Find a cut through the original network that has value equal to this maximum flow. *(2 marks)*

END OF QUESTIONS

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Candidate Signature					

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TURN OVER FOR FIGURE 1

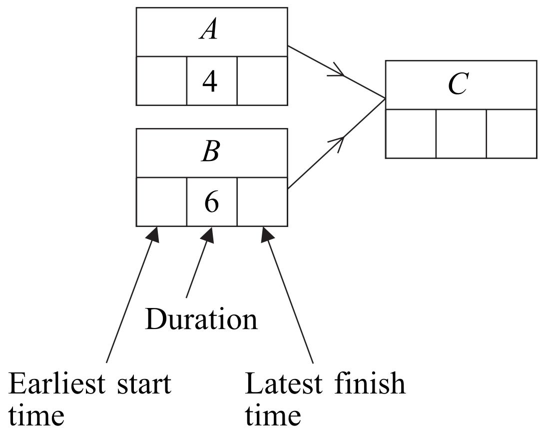
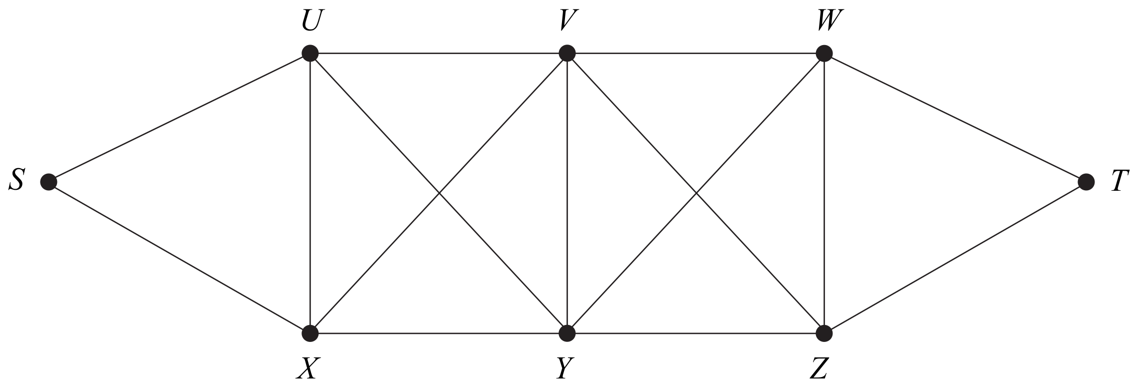


Figure 1 (for use in Question 3)



Route	Flow
SUYWT	
SXYZT	

Figure 2 (for use in Question 6)

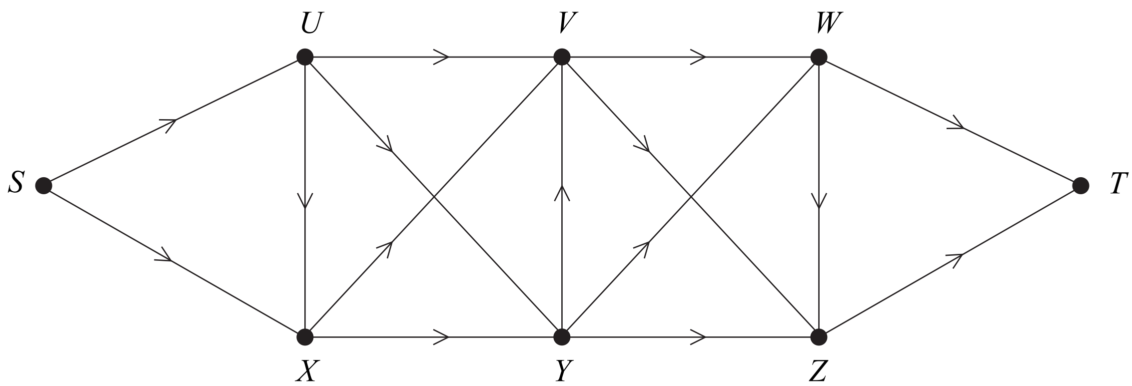


Figure 3 (for use in Question 6)

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TURN OVER FOR FIGURE 1

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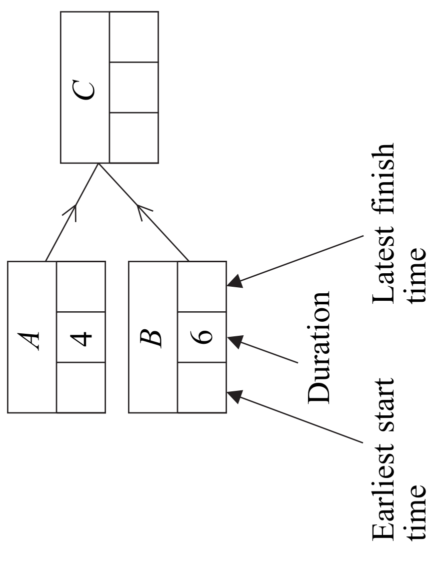
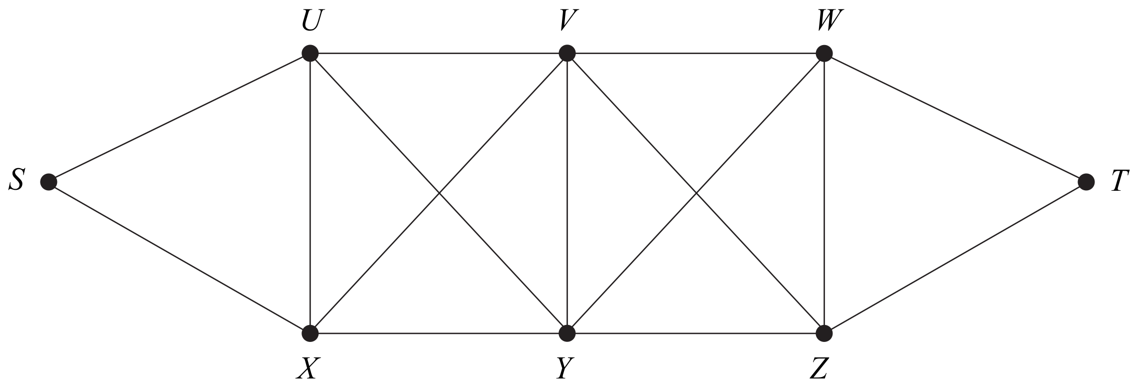


Figure 1 (for use in Question 3)



Route	Flow
SUYWT	
SXYZT	

Figure 2 (for use in Question 6)

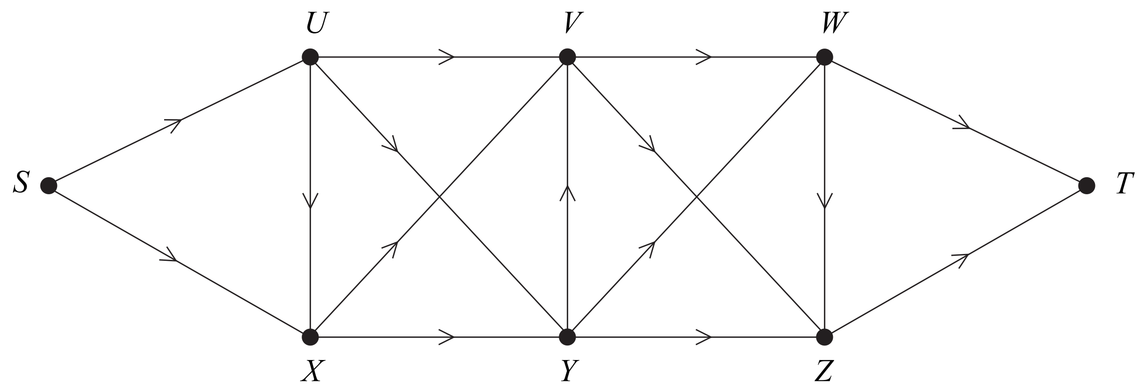


Figure 3 (for use in Question 6)

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